METHOD AND APPARATUS FOR PRODUCING AN IMAGE CONTAINING DEPTH INFORMATION

Field of the Invention

5   This invention relates to a method and apparatus for producing an image containing depth information. The invention has particular application to thermal images in the 8 to 12 micron band of the electromagnetic spectrum. However, the method and apparatus is also useful with

10   images taken in other parts of the electromagnetic spectrum (and in particular visible light), as well as electron beam images, and other forms of particle radiation images as well as acoustic images.

15   ### Background Art

Thermal images are used to produce a visual image of a scene in darkness. In such images, articles in the scene which are above ambient temperature can be easily visualised. However, one major difficulty with

20   conventional thermal images is that the images contain little or no depth information, so that it is very difficult to ascertain whether one part of the image is in front of or behind another part of the image. Thus, when a thermal image is taken of a scene which, for example,

25   may contain people, the thermal image will clearly show each of the individual people, but will not provide sufficient information to ascertain the relative positions of the people in space.

30   In many applications, it is necessary that the image does include depth information so that the image provides sufficient information of the scene to suit a particular purpose.

35   ### Summary of the Invention

The object of the invention is to provide a method and apparatus of producing images containing improved depth

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information so that a determination can be made as to the
distance of various parts of the image relative to one
another and from the location where the image is taken.

5   The invention may be said to reside in a method of
producing an image containing depth information,
comprising:
        detecting a radiation wavefield emanating from
the scene and taking at least two images of the scene at
10   different planes relative to the scene to produce a first
image comprised of a first set of intensity data values
and a second image comprised of a second set of intensity
data values;
        determining an intensity variation of data values
15   in the first set of data values relative to other data
values in the first set of data values to produce a first
set of intensity variances, and determining an intensity
variation of data values in the second set of values
relative to other data values in the second set of values
20   to produce a second set of intensity variance data;
        processing the first and second sets of intensity
variance data to obtain image data of the scene containing
depth information; and
        coding the image data which have the same depth
25   information with a code reference to identify the
different depth information in the image data.

Thus, by producing the variance data and then processing
the variance data, the variance data is able to result in
30   the production of depth information in the image which can
then be coded so that the different depth information is
apparent in the image.

In the preferred embodiment of the invention, the
35   intensity variance data is processed in accordance with
the phase image production method disclosed in our co-
pending International Patent Application Nos.

PCT/AU99/00949 and PCT/AU02/01398. The contents of these two International applications are incorporated into this specification by this reference.

5   Thus, the step of processing the intensity variance data preferably comprises:

(a)   producing a representative measure of the rate of change of variance data values of said radiation wave field over a selected surface extending generally
10   across the wave field;

(b)   producing a representative measure of said radiation wave field relating to the scene over said selected surface;

(c)   transforming said measure of rate of change
15   of variance to produce a first integral transform representation and applying to said first integral transform representation a first filter corresponding to the inversion of a first differential operator reflected in said measure of rate of change of variance to produce a
20   first modified integral transform representation;

(d)   applying an inverse of said first integral transform to said first modified integral transform representation to produce an untransformed representation;

(e)   applying a correction based on said measure
25   over said selected surface to said untransformed representation;

(f)   transforming the corrected untransformed representation to produce a second integral transform representation and applying to said second integral
30   transform representation a second filter corresponding to the inversion of a second differential operator reflected in the corrected untransformed representation to produce a second modified integral transform representation;

(g)   applying an inverse of said second integral
35   transform to said second modified integral transform representation to produce a measure of phase of said radiation wave field across said selected plane so as to

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produce said image data as phase image data containing the
depth information.

The step of producing a representative measure of said
5   radiation wavefield relating to the scene over the
selected surface may use intensity values to obtain the
representative measure or variance values.

In particular, in one embodiment, intensity values at the
10   selected surface are used.

In another embodiment, values are taken representing
maximum focus from any of the intensity data values.

15   In another embodiment, variance data values are used as a
representative measure.

In a still further embodiment, maximum variance values
taken from the sets of variance data values are used.
20
Preferably at least one of the first or second
differential operator has a form based on an optical
system used to acquire the radiation for producing the
representative measure of the rate of change of variance
25   of the radiation wavefield over the selected surface
extending generally across the wavefield.

Preferably both the first and second differential
operators have a form based on the optical system.
30
Preferably the first and second integral transforms are
produced using a Fourier transform.

In one embodiment of the invention the differential
35   operators have the form:

$$\frac{\sqrt{T_P}}{T_P + \alpha^2}$$

where,

5      $T_P(\rho) = 2\pi i \delta z \int \eta T_P^{(3)}(\rho, \eta) d\eta$

and

$$T_P^{(3)}(\rho, \eta) = \frac{i}{2\pi\rho} \left\{ \left[\left[\tfrac{1}{2} \rho_{obj}^2 (\xi^2 + 1) - \tfrac{1}{4} \rho^2 - \left(\frac{\eta}{\lambda\rho} \right)^2 - \left| \frac{\eta}{\lambda} - \tfrac{1}{2} \rho_{obj}^2 (\xi^2 - 1) \right| \right]^{\frac{1}{2}} - \left[\tfrac{1}{2} \rho_{obj}^2 (\xi^2 + 1) - \tfrac{1}{4} \rho^2 - \left(\frac{\eta}{\lambda\rho} \right)^2 - \left| \frac{\eta}{\lambda} + \tfrac{1}{2} \rho_{obj}^2 (\xi^2 - 1) \right| \right]^{\frac{1}{2}} \right\}$$

10

and wherein ρ is radial lateral spatial frequency, the η longitudinal spatial frequency and δz is the defocus distance in the plane of the object. Also

15      $\xi = \frac{NA_{condensor}}{NA_{objective}}$

where $NA_{condensor}$ and $NA_{objective}$ are respectively the numerical aperture of the condenser and the objective (These are settings and dimensions on the microscope). ρ_{obj} is the

20   maximum spatial frequency accepted by the objective.

Preferably the step of taking at least two images of a scene comprises taking the first image at a first defocused plane to produce the first image as negatively

25   focused image data and taking the second image of the scene at a second defocused plane to produce positive defocused image data, the negative and positive defocused image being taken on respective sides of a focal plane which would produce an in focus image of the scene.

30

Preferably the step of determining an intensity variance comprises determining a measure of the sharpness of each

of the data values relative to the sharpness of data
values surrounding that data value.

In the preferred embodiment of the invention the images
are captured by a charge coupled device and an intensity
value of pixels in the image is determined, and that
intensity value is compared with the intensity of
surrounding pixels in order to provide a variance value at
each pixel.

Preferably the variance is obtained by the following
equation for each pixel:

$$\text{var.}_n = \frac{1}{8} \times \sqrt{\begin{array}{l} (I_n - I_2)^2 + (I_n - I_4)^2 + (I_n - I_8)^2 \\ + (I_n - I_6)^2 + \frac{1}{\sqrt{2}}(I_n - I_1)^2 + \frac{1}{\sqrt{2}}(I_n - I_3)^2 \\ + \frac{1}{\sqrt{2}}(I_n - I_7)^2 + \frac{1}{\sqrt{2}}(I_n - I_9)^2 \end{array}}$$

wherein n is the particular pixel, and the values 1 to 9
represent the pixels which surround that pixel in an array
of pixels.

Thus, in this embodiment, each pixel is compared with
pixels which immediately surround the particular pixel in
order to determine the variance. In other embodiments,
each pixel could be compared with the pixels which
surround that pixel, together with the pixels which
surround the pixel and are one further pixel away and so
on. The variance could also be determined by other
methods so that a variance of each pixel relative to the
other pixels in each data set is obtained.

Preferably the step of coding parts of the image data
comprises applying different colours to parts of the image
which have the same depth information so as to produce a
visual image in which the relative distance of parts of
the scene compared to one another can be determined.

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Preferably a third image of the scene is taken at an in focus plane to produce a third set of intensity data values and the third set of intensity data values are
5   overlaid with the coded image data to produce a visual image containing structural information of the scene as well as the different depth information in the scene.

However, in other embodiments, rather than providing a
10   visual image, the image could be an electronic image stored in a computer, and the code reference could simply be a code value allocated by the computer to the different depth information so that if images at a particular location are required, information as to those parts of
15   the image can be extracted by the computer by the code reference.

Preferably the different depth information is provided by allocated a grey scale value to pixel values in the image.
20
The invention may also be said to reside in a computer program for performing the above-mentioned method.

The invention may also be said to reside in an apparatus
25   for producing an image containing depth information comprising:
             a camera for detecting a radiation wavefield emanating from a scene and taking at least two images of the scene at different planes relative to the scene to
30   produce a first set of intensity data values and a second set of intensity data values;
             a processor for determining an intensity variation of data values in the first set of data values compared to other data values in the first set of data
35   values to produce a first set of variance data values, and for determining a variance of one data value in the second set of data values compared to other data values in the

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second set of data values to produce a second set of
variance data values;

      the processor also being for processing the first
and second variance data values to produce image data of
the scene containing depth information; and

      the processor also being for coding parts of the
image which have the same depth information with a code
reference to identify the different depth information in
the image.

Thus, the processor for determining an intensity variation
is a processor for:

      (a)   producing a representative measure of the
rate of change of variance data values of said radiation
wave field over a selected surface extending generally
across the wave field;

      (b)   producing a representative measure of said
radiation wave field relating to the scene over said
selected surface;

      (c)   transforming said measure of rate of change
of variance to produce a first integral transform
representation and applying to said first integral
transform representation a first filter corresponding to
the inversion of a first differential operator reflected
in said measure of rate of change of variance to produce a
first modified integral transform representation;

      (d)   applying an inverse of said first integral
transform to said first modified integral transform
representation to produce an untransformed representation;

      (e)   applying a correction based on said measure
over said selected surface to said untransformed
representation;

      (f)   transforming the corrected untransformed
representation to produce a second integral transform
representation and applying to said second integral
transform representation a second filter corresponding to
the inversion of a second differential operator reflected

in the corrected untransformed representation to produce a
second modified integral transform representation;

(g)   applying an inverse of said second integral
transform to said second modified integral transform
5   representation to produce a measure of phase of said
radiation wave field across said selected plane so as to
produce said image data as phase image data containing the
depth information.

10   The step of producing a representative measure of said
radiation wavefield relating to the scene over the
selected surface may use intensity values to obtain the
representative measure or variance values.

15   In particular, in one embodiment, intensity values at the
selected surface are used.

In another embodiment, values are taken representing
maximum focus from any of the intensity data values.
20

In another embodiment, variance data values are used as a
representative measure.

In a still further embodiment, maximum variance values
25   taken from the sets of variance data values are used.

Preferably at least one of the first or second
differential operator has a form based on an optical
system used to acquire the radiation for producing the
30   representative measure of the rate of change of variance
of the radiation wavefield over the selected surface
extending generally across the wavefield.

Preferably both the first and second differential
35   operators have a form based on the optical system.

Preferably the first and second integral transforms are

produced using a Fourier transform.

In one embodiment of the invention the differential operators have the form:

$$\frac{\sqrt{T_P}}{T_P + \alpha^2}$$

where,

$$T_P(\rho) = 2\pi i \delta z \int \eta T_P^{(3)}(\rho, \eta) d\eta$$

and

$$T_P^{(3)}(\rho, \eta) = \frac{i}{2\pi\rho} \left\{ \left[\frac{1}{2}\rho_{obj}^2(\xi^2 + 1) - \frac{1}{4}\rho^2 - \left(\frac{\eta}{\lambda\rho}\right)^2 - \left|\frac{\eta}{\lambda} - \frac{1}{2}\rho_{obj}^2(\xi^2 - 1)\right| \right]^{\frac{1}{2}} - \left[\frac{1}{2}\rho_{obj}^2(\xi^2 + 1) - \frac{1}{4}\rho^2 - \left(\frac{\eta}{\lambda\rho}\right)^2 - \left|\frac{\eta}{\lambda} + \frac{1}{2}\rho_{obj}^2(\xi^2 - 1)\right| \right]^{\frac{1}{2}} \right\}$$

and wherein ρ is radial lateral spatial frequency, the η longitudinal spatial frequency and δz is the defocus distance in the plane of the object. Also

$$\xi = \frac{NA_{condensor}}{NA_{objective}}$$

where $NA_{condensor}$ and $NA_{objective}$ are respectively the numerical aperture of the condenser and the objective (These are settings and dimensions on a microscope, if a microscope is used in the capture of the various images by camera). ρ_{obj} is the maximum spatial frequency accepted by the objective.

If images are captured through a different optical system such as a telescope or simply by the optics of a camera, then the above operators are modified to suit the various optics of that system.

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Preferably the step of taking at least two images of a
scene comprises taking a first image at a first defocused
plane to produce a negatively focused image and taking a
5   second image of the scene at a second defocused plane to
produce a positive defocused image, the negative and
positive defocused image being taken on respective sides
of a focal plane which would produce an in focus image of
the scene.
10

Preferably the step of determining an intensity variance
comprises determining a measure of the sharpness of each
of the data values relative to the sharpness of data
values surrounding that data value.
15

In the preferred embodiment of the invention the images
are captured by a charge coupled device and an intensity
value of pixels in the image is determined, and that
intensity value is compared with the intensity of
20   surrounding pixels in order to provide a variance at each
pixel.

Preferably the variance is obtained by the following
equation for each pixel:
25

$$\text{var.}_n = \frac{1}{8} \times \sqrt{\begin{array}{l} (I_n - I_2)^2 + (I_n - I_4)^2 + (I_n - I_8)^2 \\ + (I_n - I_6)^2 + \frac{1}{\sqrt{2}}(I_n - I_1)^2 + \frac{1}{\sqrt{2}}(I_n - I_3)^2 \\ + \frac{1}{\sqrt{2}}(I_n - I_7)^2 + \frac{1}{\sqrt{2}}(I_n - I_9)^2 \end{array}}$$

wherein n is the particular pixel, and the values 1 to 9
represent the pixels which surround that pixel in an array
of pixels and I is the intensity value of the respective
30   pixel.

Preferably the step of coding parts of the processed image
comprises applying different colours to parts of the image

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which have the same depth information so as to produce a
visual image in which the relative distance of parts of
the scene compared to one another can be determined.

5   However, in other embodiments, rather than providing a
visual image, the image could be an electronic image
stored in a computer, and the code reference could simply
be a code value allocated by the computer to the different
depth information so that if images at a particular
10   location are required, information as to those parts of
the image can be extracted by the computer by reference to
the code reference.

Preferably the different depth information is provided by
15   allocated a grey scale value to pixel values in the image.

Brief Description of the Drawings
A preferred embodiment of the invention will be described
with reference to the accompanying drawings in which:
20   Figure 1 is a diagram to assist in illustrating
the preferred embodiment of the invention;
Figure 2 is a view of an apparatus embodying the
invention;
Figure 3 is a diagram showing pixels of a camera
25   used to take the images in the preferred embodiment of the
invention;
Figure 4 is a diagram illustrating the production
of different depth information;
Figure 5 is a diagram illustrating how different
30   depth information is coded according to the preferred
embodiment of the invention;
Figure 6 is a flowchart showing an implementation
of the method of the preferred embodiment;
Figure 7 is a schematic illustration of an
35   arrangement which uses a microscope in the capture of
images;
Figure 8 is a schematic drawing of an exemplary

system according to one embodiment of the invention; and
Figure 9 is a flow chart relating to one
embodiment of the invention.

5   Detailed Description of the Preferred Embodiment
    The preferred embodiment of the invention effectively
    processes images in accordance with the phase
    determination technique disclosed in our International
    Applications PCT/AU99/00949 and PCT/AU02/01398. However,
10 before the algorithm disclosed in these applications is
    used to process the data relating to the captured scenes
    to produce the image, the data is first manipulated so as
    to produce a variance data so the variance data is
    processed in accordance with the algorithms rather than
15 pure intensity data, as disclosed in the above
    International applications. However, apart from this, the
    method of processing the data is the same as that
    disclosed in the above International applications.

20 Figure 1 is a diagram showing an image of a scene at three
    different focal planes. The plane I_0 is an in focus plane
    which will be used to capture an in focus image of the
    scene. The scene may include a number of articles or
    objects such as vehicles, personnel, or the like, and in
25 general, the preferred embodiment of the invention is to
    be used for obtaining thermal images. However, the
    invention could be used with other types of images and
    improve the depth information in those images. The
    invention has particular application to thermal images
30 because thermal images generally contain little or no
    depth information which enables different parts of the
    images to be compared in terms of distance with other
    parts of the image. Thus, in thermal images, it is very
    difficult to ascertain whether one part of the image is
35 closer to or further away from the location from where the
    images were captured, or where parts of the image are
    located in space relative to one another.

The plane V_- is a negatively defocused plane within the
depth of field of the camera 20 (see Figure 2) used to
capture the images, the plane I_0 is the focal plane, and
5   the plane V_+ is a positively defocused plane also within
the depth of field of the camera 20, but positively
defocused relative to the plane I_0. In other words, the
plane V_- is closer to the scene than the in focus plane I_0
and the plane V_+ is further away from the scene than the in
10   focus plane I_0.

Typically, parts of the image taken at the plane V_- which
happen to be in focus t the plane V_- will produce a very
sharp image point 21 at the plane V_-, but will be a very
15   fuzzy defocused image at the plane V_+ as shown by reference
22. The waveform which produces the images at 21 and 22
is shown at 23 in Figure 1. Similarly, part of the image
24 which is defocused and blurry at the plane V_- may be
focused at the plane V_+ as represented by reference 25.
20   Once again, the form of the wavefield producing these
parts of the images is shown at 26.

As is shown in Figure 2, the camera 20 is used to capture
images of the scene schematically shown at S in Figure 2
25   at the planes V_-, I_0 and V_+ so that three sets of intensity
data at those three planes is captured by the pixels of
the camera 20.

In the preferred embodiment of the invention, the camera
30   20 is a cryogenically-cooled, 640 x 480 pixel array
thermal camera operating in the 8 to 12 micron band with
adjustable-focus Ge lens. The intensity data captured by
the camera 20 is processed in a processor 40 and images
may be displayed on a display 42.
35

The processor 40 first determines a variance value for
each of the pixels for each of the two images at the

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planes V₋ and V₊. Thus, two sets of variance data is
therefore produced.

5   Figure 3 shows an example of nine pixels of the pixel
array forming camera 20 and the variance at the pixel I_5 is
determined in accordance with the following equation:

$$\text{var.}_5 = \frac{1}{8} \times \sqrt{\begin{aligned} & (I_5 - I_2)^2 + (I_5 - I_4)^2 + (I_5 - I_8)^2 \\ & + (I_5 - I_6)^2 + \frac{1}{\sqrt{2}}(I_5 - I_1)^2 + \frac{1}{\sqrt{2}}(I_5 - I_3)^2 \\ & + \frac{1}{\sqrt{2}}(I_5 - I_7)^2 + \frac{1}{\sqrt{2}}(I_5 - I_9)^2 \end{aligned}}$$

10   Thus, a variance value for the fifth pixel in the array of
Figure 3 is determined. In Figure 3, pixels which are
arranged at the edge of the array can be set to a value 0
so that they are not included in the calculations. This
reduces the size of the image slightly by 2 pixel widths.
15   However, it makes calculation much easier because all of
the variances can be determined in the above mentioned
manner because all pixels will then be surrounded by 8
other pixels. Similarly, variance values for each of the
other pixels in the array is determined for each of the
20   image pixels of planes V₋ and V₊ so that two sets of
variance data is produced. The two sets of variance data
is then used in the algorithm to produce the phase image
data of the scene instead of the pure intensity data
originally captured by the camera 20. The use of the
25   variance data provides a measure of the sharpness of the
image at each of the planes V₋ and V₊ and therefore,
inherently includes a measure of the depth of parts of the
image relative to other parts of the image. The variance
data is then processed in the same manner as the intensity
30   data, as disclosed in the aforementioned International
applications, so as to produce a phase image of the scene
in which depth of field information is included. Because
the variance data is used to produce the phase image, the

image will contain very little detail of the actual
structure of articles or things in the scene, but will
contain general outlines showing features of the scene.
The depth of position information in the processed image
5   can be given by a grey scale value at each of the pixels
in the processed image data, which in turn therefore
provide a relative measure of the part of the scene which
relates to each pixel relative to other parts of the
scene, as well as relative to the position of parts of the
10   scene from where the images were taken. For example, as
is best shown in Figure 4, something which is very close
to where the image is taken appears darker, as shown by
reference 61, and something which is further away appears
much lighter as at 62.

15

The grey scale values allocated to each of the pixels in
the processed phase image can be used to provide an actual
measure of the distance of the parts of the scene from
where the image was taken and relative to one another.
20   For example, if the optics of the camera is known and the
distance between the planes V_1 and V_+ is also known, actual
measurement values can be associated with various grey
scale values in the image to provide a distance measure of
parts of the scene relative to one another, and also from
25   where the images were taken.

The image shown in Figure 4 can then be coded so as to
more clearly bring out the different distances of articles
in the image by applying a code reference to parts of the
30   image which have the same grey scale value. This can be
done in a variety of different ways, but most preferably
is performed by coding parts of the image with a false
colour palette so that parts of the image which have the
same grey scale value are coloured with the same colour.
35   This therefore makes it very much easier to perceive depth
information in the image than in the mere conventional
thermal image in which the entire image appears to be

approximately the same shade. For instance, in Figure 5,
the grey scale values are shown for near articles in the
image which may be say 10 metres from where the image is
taken, up to 100 metres away. Those grey scale values can
5   be allocated with different colours in the manner referred
to above, so that a particular colour can be associated
with a particular distance in the image to make it
therefore much more easy to perceive where articles in the
image are actually located relative to one another.

10

In order to produce an image in which the details of the
article are shown, rather than the pure distance
information, the processed image can then be applied to
the intensity image taken at plane I_0 which is the in focus

15 plane, to give both the intensity information of the image
and the depth information of the image in the one image.
Thus, the intensity information provides the structural
detail of the various components in the image and the
depth information, as previously described, enables you to

20 ascertain the relative position of parts of the image.
Thus, this image will not only contain the depth
information, but will also make it much easier to
ascertain what various components of the image are,
because the actual structure in the image will be seen.

25

Thus, according to the preferred embodiment of the
invention, the imaging range of the lens on the camera 20
and the colour rendering can be calibrated and therefore
positions of the image can be quantified. This technique

30 then offers the opportunity to provide a passive ranging
capability. In conditions of near thermal uniformity,
where there is very little contrast in the field of view,
the depth positional information provided by this process
suggests another contrast mechanism for viewing and

35 processing images of scenes. Finally, under obscured
viewing conditions, such as under camouflage, the outline
of an object can be disrupted, making identification

difficult. The colour depth rendering of the scene allows
the piecing together of image components at common
positions, as even though the outlines can be disguised,
the physical positions in space cannot be.

5

The manner in which the algorithm operates to process the
variance data to produce the phase image is described
below.

10   At each point in space, an optical beam possesses two
properties: *intensity* and *phase*. Intensity is a measure
of the amount of energy flowing through each point, while
phase gives a measure of the direction of the energy flow.

15   Intensity may be measured directly, for example by
recording an image on film. Phase is typically measured
using interference with a "reference beam". In contrast
the present method gives a non-interferometric method for
measuring phase.

20

Intensity can be measured over two parallel planes V-, V+
extending across the direction of propagation of the wave
field on the side remote from the incident radiation.

25   The present invention determines phase by providing a
solution to the transport-of-intensity equation:

$$(1) \qquad \nabla_{\perp} \bullet (I\nabla_{\perp}\phi \ ) = -k\frac{\partial I}{\partial z}$$

30   where I is the intensity in the plane, the gradient
operator in the plane is denoted $\nabla_{\perp}$, k is the wave number
of the radiation, and $\partial I/\partial z$ is the intensity derivative or
rate of change of intensity. However, in order to obtain

the depth information which is previously referred to,
rather than using the intensity values which are obtained
by the camera at the two planes, the variance values
calculated in the above mentioned manner are obtained so
5   that a variance derivative, or rate of change of variance
$\partial V/\partial z$. Note that $\partial V/\partial z$ is estimated from the difference of
the measurements in the planes V_- and V_+ shown in Figure 1,
while the intensity I is given by the average of the
measurements.

10

In order to obtain a solution to equation 1 the function A
is first defined as:

$$(2) \qquad \nabla_\perp A = V \nabla_\perp \phi \ .$$

15   Thus equation (1) becomes:

$$(3) \qquad \nabla_\perp \bullet (\nabla_\perp A) = -k \partial_z V \ .$$

Making use of a standard identity $\nabla_\perp \bullet \nabla_\perp = \nabla_\perp^2$, this may be
20   written:

$$(4) \qquad \nabla_\perp^2 A = -k \partial_z V$$

where $\nabla_\perp^2$ denotes the two-dimensional Laplacian acting
25   over the surface of the image. This equation has the
following symbolic solution:

$$(5) \qquad A = -k \nabla_\perp^{-2} \partial_z V \ .$$

30   If the gradient operator $\nabla_\perp$ is applied to both sides of
this equation, it becomes:

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$$(6) \qquad \nabla_\perp A = -k\nabla_\perp \nabla_\perp^{-2} \partial_z V \ .$$

The defining equation (2) for the function A allows (6) to
be transformed into:

$$(7) \qquad I\nabla_\perp \phi = -k\nabla_\perp \nabla_\perp^{-2} \partial_z V \ .$$

Dividing both sides by I then yields:

$$(8) \qquad \nabla_\perp \phi = -kI^{-1}\nabla_\perp \nabla_\perp^{-2} \partial_z V \ .$$

However, as is mentioned above, rather than dividing both
sides by the intensity value I, a maximum focus value
across all of the images could be used, or the variance
data at the plane could be used, or a maximum variance
across all of the variance state of values could be used.

Taking the two dimensional divergence $\nabla_\perp \bullet$ of both sides of
(8), and again making use of the standard identity
$\nabla_\perp \bullet \nabla_\perp = \nabla_\perp^2$, then (8) becomes:

$$(9) \qquad \nabla_\perp^2 \phi = -k\nabla_\perp \bullet \left[I^{-1}\nabla_\perp \nabla_\perp^{-2} \partial_z V \right].$$

This equation has the following symbolic solution:

$$(10) \qquad \phi = -k\nabla_\perp^2 \left(\nabla_\perp \bullet \left[I^{-1}\nabla_\perp \nabla_\perp^{-2} \partial_z V \right] \right).$$

In order to implement a practical solution to equation
(10), the following formulae are required. A suitably-

well-behaved function f(x,y) may be written in the form of
a two-dimensional Fourier integral:

$$(11) \quad f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{f}(k_x,k_y) e^{i(k_x x + k_y y)} dk_x dk_y.$$

The function $\hat{f}(k_x,k_y)$ is called the "Fourier transform" of
f(x,y).

The x derivative of (11) yields:

$$(12) \quad \frac{\partial}{\partial x} f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ik_x \hat{f}(k_x,k_y) \right] e^{i(k_x x + k_y y)} dk_x dk_y.$$

Hence the Fourier transform of $\frac{\partial}{\partial x}$ f(x,y) is equal to the

Fourier transform of f(x,y) multiplied by ik_x. Stated

differently, $\frac{\partial}{\partial x} = i\mathbf{F}^{-1}k_x\mathbf{F}$, where **F** denotes Fourier

transformation and $\mathbf{F}^{-1}$ denotes inverse Fourier
transformation. Similar considerations apply to
$\frac{\partial}{\partial y}$ f(x,y).

If the Laplacian $\nabla_{\perp}^{2} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ of (11) is obtained and

similar reasoning applied, it follows that $\nabla_{\perp}^{-2} = -\mathbf{F}^{-1}k_r^{-2}\mathbf{F}$,

where $k_r^2 = k_x^2 + k_y^2$. Thus:

$$(13) \quad \nabla_{\perp}^{-2} = -\mathbf{F}^{-1}k_r^{-2}\mathbf{F}, \quad \frac{\partial}{\partial x} = i\mathbf{F}^{-1}k_x\mathbf{F}, \quad \frac{\partial}{\partial y} = i\mathbf{F}^{-1}k_y\mathbf{F}.$$

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Here, F denotes Fourier transformation, F^{-1} denotes inverse
Fourier transformation, $(k_x \ k_y)$ are the Fourier variables
conjugate to (x,y), and

5
$$k_r^2 = k_x^2 + k_y^2 \ .$$

Equations (13) can be used to rewrite equation (10) in the
form

10   (14)

$$\phi = \phi^{(x)} + \phi^{(y)}, \begin{cases} \phi^{(x)} = F^{-1} k_r^{-2} k_x F I^{-1} F^{-1} k_x k_r^{-2} F \left[k \dfrac{\partial V}{\partial z} \right] \\[4mm] \phi^{(y)} = F^{-1} k_r^{-2} k_y F I^{-1} F^{-1} k_y k_r^{-2} F \left[k \dfrac{\partial V}{\partial z} \right] \end{cases}$$

In practice division by intensity is only performed if
that intensity is greater than a certain threshold value
15   (eg. 0.1% of the maximum value).

Division by k_r does not take place at the point kr = 0 of
Fourier space;
instead multiplication by zero takes place at this point.
20   This amounts to taking the Cauchy principal value of the

integral operator $\nabla_\perp^{-2}$.

In order to quantitatively measure the phase of object it
is necessary to incorporate some physical constants into
25   the phase recovery algorithm given in Equation (14)
relating to the experimental setup in use to quantify the
variables k_x, k_y. This can be done by rewriting equation

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(14) in the following form suitable for implementation using a fast Fourier transform:

$$\phi = \phi_x + \phi_y, \begin{cases} \phi_x = -\dfrac{2\pi}{\lambda \delta z} \dfrac{1}{(N\Delta x)^2} \mathbf{F}^{-1} \dfrac{i}{i^2+j^2} \mathbf{F} \dfrac{1}{I(x,y)} \mathbf{F}^{-1} \dfrac{i}{i^2+j^2} \mathbf{F}\{V_+ - V_-\} \\[2ex] \phi_y = -\dfrac{2\pi}{\lambda \delta z} \dfrac{1}{(N\Delta x)^2} \mathbf{F}^{-1} \dfrac{j}{i^2+j^2} \mathbf{F} \dfrac{1}{I(x,y)} \mathbf{F}^{-1} \dfrac{j}{i^2+j^2} \mathbf{F}\{V_+ - V_-\} \end{cases}$$

where $i,j \in \left[\dfrac{-N}{2}, \dfrac{N}{2} \right]$ index the frequent components of $F(V_+ - V_-)$

where the intensity derivative $\partial_z V(x,y)$ is obtained by subtracting two images V_+ and V_- separated by a distance δz, i and j are the pixel numbers on the image, and using the fact that the Fourier space step size is given by

$$\Delta k = \frac{1}{N\Delta x}$$

where the image is an **N x N** array of pixels of size Δx. Thus in addition to measuring the three intensity distributions it is necessary to know the pixel size Δx, defocus distance δz and wavelength λ in order to make a quantitative phase measurement. All of these quantities can be readily determined: the pixel size can be determined directly for example from the CCD detector geometry (in the case of direct imaging), or by existing techniques for calibrating the transverse distance scales (in the case of an imaging system), the defocus distance can be measured directly, and the spectral distribution of the illumination can be determined either by monochromating the incident field or by analysing the spectral distribution of the radiation using existing spectroscopic methods.

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An example of the phase-retrieval method implementing the
solution of equation (14) can be represented by the
flowchart shown in Figure 6. As shown in Figure 2 the
5   quantitative determination of phase of a radiation wave
field proceeds from a set of variance measurements $\{V_n\}$
over the two spaced apart planes V_- and V_+. A measurement
of central intensity $I(x,y)$ in a selected plane parallel
to and midway between the planes V_- and V_+ is also
10   obtained. The intensity measurements are performed over a
defined array on each of the two planes A and B and the
respective values subtracted to produce a measure of the
intensity derivative. This value is multiplied by the
negative of the average wave number. The data are split
15   into two component sets and a fast Fourier transform is
performed to produce the respective x and y components in
the Fourier domain. A filter is then applied to the
Fourier domain representations to correspond to the
inversion of a differential operator reflected in the
20   untransformed representation. An inverse Fourier
transform is then performed on each of the x and y
components to produce a spatial domain value from which
the differential operator has been removed. A division by
the central intensity $I(x,y)$ obtained by averaging the
25   intensity measurements over planes V_+ and V_- is then
performed if the intensity level is above a selected
threshold level. The resultant data is again Fourier
transformed and multiplied by the same filter to again
correspond to the inversion of a differential operator
30   reflected in the untransformed data. The resultant
components are again inverse Fourier transformed and
summed to provide a retrieved phase measurement.

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It will be apparent that in general the method according to this invention can proceed from any suitable representative determination of intensity derivative or rate of change of intensity over a selected surface
5   extending across the propagation direction and the intensity over that same surface. As will be explained in various examples these data can be obtained in a variety of ways and the method implemented to yield phase of the radiation wave field.
10

Rewriting equation (14) with:

$$\Omega_x(k_x, k_y, \alpha) = k_x k_r^{-2}$$

15

$$\Omega_y(k_x, k_y, \alpha) = k_y k_r^{-2}$$

$$\phi(x, y) = \phi^{(x)}(x, y) + \phi^{(y)}(x, y),$$

gives

20

(15)

$$
\begin{cases}
\phi^{(x)}(x, y) = \mathbf{F}^{-1}\Omega_x(k_x, k_y, \alpha)\mathbf{F}\dfrac{1}{I(x,y)}\mathbf{F}^{-1}\Omega_x(k_x, k_y, \alpha)\mathbf{F}\left[\overline{k}\dfrac{\partial V}{\partial z}\right]. \\[4mm]
\phi^{(y)}(x, y) = \mathbf{F}^{-1}\Omega_y(k_x, k_y, \alpha)\mathbf{F}\dfrac{1}{I(x,y)}\mathbf{F}^{-1}\Omega_y(k_x, k_y, \alpha)\mathbf{F}\left[\overline{k}\dfrac{\partial V}{\partial z}\right]
\end{cases}
$$

where:

25           $\phi(x, y)$ denotes the recovered phase,

           F denotes Fourier transformation, and $\mathbf{F}^{-1}$ denotes inverse Fourier transformation,

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$I(x,y)$ is the intensity distribution over the plane of interest,

(x,y) are Cartesian coordinates over the plane of interest,

(k_x, k_y) are the Fourier variables conjugate to (x,y)

$\overline{k} = 2\pi/\overline{\lambda}$ is the average wave number of the radiation,

$\overline{\lambda}$ is the average wavelength of the radiation,

$\partial V/\partial z$ is the estimate for the longitudinal variance derivative,

α is the regularization parameter used to stabilize the algorithm when noise is present.

As given above, the solution to the transport of intensity equation (1) assumes a perfect imaging system. That is, there are no "aberrations" present in the optical system used to obtain the intensity data which is fed into the algorithm. Of course, no imaging system is perfect. The imperfections present in an imaging system may be quantified by a set of numbers:

(16) $A_1, A_2, A_3, ...$

which are termed aberration coefficients.

If intensity data were taken on an imperfect instrument whose imperfections were characterized by a certain set of known aberration coefficients $A_1, A_2, A_3, ...$, it would be desirable if the filters $\Omega_x(k_x k_y, \alpha)$ and $\Omega_y(k_x k_y, \alpha)$ present in (15) could be replaced by modified filters which explicitly depend upon the aberration coefficients:

(17) $\tilde{\Omega}_x(k_x k_y, \alpha, A_1, A_2, A_3,...)$ and $\tilde{\Omega}_y(k_x k_y, \alpha, A_1, A_2, A_3,...)$

This would allow the imperfections of the imaging system
to be explicitly taken into account, leading to
quantitatively correct phase retrieval using aberrated
imaging systems. For the special case of a non-absorbing
phase object in a radiation wave field of uniform
intensity with weak (i.e. much less than 2π radians) phase
variations the appropriate modified filters lead to the
following functional form for the phase-retrieval
algorithm:

$$(18) \qquad \phi(x,y) = \mathbf{F}^{-1}\left\{\frac{\mathbf{F}\left\{V_{aberrated}(x,y)-1\right\}}{\left(-2\pi.\delta z.\bar{\lambda}(k_x^2 + k_y^2) - 2\sum_m \sum_n A_{mn}k_x^m k_y^n\right)}\right\},$$

where:

$V_{aberrated}(x,y)$ is the aberrated variance measured at
defocus distance δz,

A_{mn} are the aberration coefficients which
characterize the imperfect imaging system.

If a filter is defined:

$$(19) \qquad \tilde{\Omega}(k_x k_y, \alpha, A_1, A_2, A_3,...) =$$

$$\frac{1}{\sqrt{-2\pi.\delta z.\bar{\lambda}(k_x^2 + k_y^2) - 2\sum_m \sum_n A_{mn}k_x^m k_y^n}}$$

Then (18) becomes:

$$(20) \qquad \phi(x,y) = \mathbf{F}^{-1}\tilde{\Omega}\mathbf{F}\frac{1}{I_0}\mathbf{F}^{-1}\tilde{\Omega}\mathbf{F}\{I_{aberrated}(x,y) - 1\}$$

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The term $\{ I_{aberrated}(x,y)-1 \}$ is a measure of rate of change of
intensity. I_0 intensity is a measurable constant for
uniform intensity so that (20) is the same general form as
(15). Consequently the special case of aberration can be
5   dealt with by changing the filter in the general method
described above. The x and y component filters Ω_x and Ω_y
are given by

$$(21) \qquad \Omega_x = \Omega_y = \frac{1}{\sqrt{2}}\widetilde{\Omega}$$

10

Figure 7 schematically shows an arrangement for
quantitative phase amplitude microscopy. A sample is
illuminated using a source of white light Köhler
illumination 15, commonly found on optical microscopes.
15   The light is transmitted through an object 16 and
collected by a microscope imaging system 17 and relayed to
a CCD camera 18 or other digital imaging device having a
planar imaging surface. Three images are collected: an
in-focus image, I_o, and two slightly out of focus images I_+
20   and I_-. The defocus is obtained by suitable means such as a
drive system 19 to adjust the microscope focus knob. The
defocus introduced is usually quite small so that
degradation in spatial resolution is minimised, although
the optimal amount of defocus to use is determined by
25   sample properties and imaging geometry such as
magnification, numerical apertures, etc.

When taking the images the numerical aperture of the
condenser is chosen to be less than the numerical aperture
30   of the objective being used. If this is not the case then
serious image degradation will occur, although the precise

amount by which the condenser and objective numerical apertures should differ involves a tradeoff between image fidelity and spatial resolution, with the optimal difference depending on the sample properties and the
5   optics used.

The variance data values are determined in the manner previously described from the intensity values collected at the planes V_+ and V_- and are subtracted to produce a
10   representative measure of rate of change of variance (variance derivative). From this and the intensity data of collected image I_o the method described above can be used to produce quantitative information about the magnitude of the phase shift in the image plane.
15
There may be cases in which it is desirable to take more than two images in order to obtain a better estimate of the intensity derivative dV/dz. A function can then be fitted to this data from which dV/dz can be computed and
20   used in the phase determination method in place of the simple subtraction of two images normally used.

It is also possible to operate this system in reflection geometry to obtain surface topography. The principle of
25   operation is the same, however the optics have to be folded back on themselves to form a reflection geometry - otherwise the process is identical.

For certain applications it can also be desirable to
30   filter the light to a particular wavelength, although this is not necessary for the described imaging process as it works equally well with white light.

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An implementation is shown in Figure 8. An Olympus BX-60
optical microscope 20 was equipped with a set of UMPlan
metallurgical objectives and a universal condenser to
provide Köhler illumination. In order to be able to
5   compare the results with existing imaging modes Nomarski
DIC optics and a set of cover-slip corrected UplanApo
objectives were also acquired for this microscope,
enabling images to be taken of the same field of view
using both phase retrieval and Nomarski DIC for the
10   purposes of qualitative comparison. A 12-bit scientific
grade Photometrics SenSys CCD camera 21 equipped with a
1300x1035 pixel Kodak KAF-1400 CCD chip was added to the
0.5x video port on the microscope to acquire digital
images of the sample.

15

The phase recovery technique of this embodiment of the
invention requires the acquisition of defocused images. A
stepper motor drive system 22 was attached to the focus
knob of the microscope. This stepper motor 22 was
20   attached to the parallel port of a 133MHz Pentium PC 23
also used to control the CCD camera 21, enabling full
automation of the acquisition of through-focus image
sequences. This data acquisition system was linked to
custom software written to recover phase images from the
25   CCD images, thereby enabling full automation of the image
acquisition and data processing sequences.

The form of the differential operators used in the
preferred embodiment of this invention are based on the
30   optics of the system used to obtain the above-mentioned
images. Thus, the operator takes into account the details
of the optical system used to take the images. This is
achieved by:

Determine the numerical aperture of the condenser, $NA_{condensor}$

Determine NA of objective, $NA_{objective}$, and $\rho_{objective}$, the radius of the objective aperture

$$\xi = \frac{NA_{condensor}}{NA_{objective}}$$

(These are settings and dimensions on the microscope.)

Determine radial lateral spatial frequency, ρ, and longitudinal spatial frequency, η.

(These are dependent on the pixelation and position distribution of images taken in the series.)

Determine the wavelength, λ, of the radiation to be used.

The new form of the operator is

$$\frac{\sqrt{T_P}}{T_P + \alpha^2}$$

where,

$$T_P(\rho) = 2\pi i \delta z \int \eta T_P^{(3)}(\rho, \eta) d\eta$$

and

$$T_P^{(3)}(\rho, \eta) = \frac{i}{2\pi\rho} \left\{ \left[\frac{1}{2}\rho_{obj}^2 (\xi^2 + 1) - \frac{1}{4}\rho^2 - \left(\frac{\eta}{\lambda\rho}\right)^2 - \left|\frac{\eta}{\lambda} - \frac{1}{2}\rho_{obj}^2 (\xi^2 - 1)\right| \right]^{\frac{1}{2}} - \left[\frac{1}{2}\rho_{obj}^2 (\xi^2 + 1) - \frac{1}{4}\rho^2 - \left(\frac{\eta}{\lambda\rho}\right)^2 - \left|\frac{\eta}{\lambda} + \frac{1}{2}\rho_{obj}^2 (\xi^2 - 1)\right| \right]^{\frac{1}{2}} \right\}$$

Figure 9 is a flow chart generally illustrating how T_ρ is

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determined by means of the above equation merely showing
breakdown of the various components of the equation.

The calculator depth image can then be presented in
5   different modalities to give better visualisation. Such
modalities including DIC, bright field images, dark field
images and other conventional modality images. Techniques
for forming these types of images from the phase data
determined in accordance with the present invention are
10   described in our copending International Application No.
PCT/AU02/00590 (the contents of which is incorporated into
this specification by this reference).

Since modifications within the spirit and scope of the
15   invention may readily be effected by persons skilled
within the art, it is to be understood that this invention
is not limited to the particular embodiments described by
way of example hereinabove.

20   In the claims which follow and in the preceding
description of the invention, except where the context
requires otherwise due to express language or necessary
implication, the word "comprise", or variations such as
"comprises" or "comprising", is used in an inclusive
25   sense, ie. to specify the presence of the stated features
but not to preclude the presence or addition of further
features in various embodiments of the invention.